Answer Refinement Modification

Refinement Type System for Algebraic Effects and Handlers

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Fuga Kawamata
Waseda University

Hiroshi Unno
University of Tsukuba

Taro Sekiyama
National Institute of Informatics

Tachio Terauchi
Waseda University

Algebraic Effects and Handlers

- A mechanism to structure programs with effects in a modular way
- Can represent many kinds of effects
  - exception, state, I/O, nondeterminism, concurrency, etc.

```
handle
  Set 1; let a1 = Get () in
  Set 2; let a2 = Get () in
  a1 + a2
with
  | return x   -> λs. x
  | Set  v, k -> λs. k () v
  | Get (), k -> λs. k  s s
```

```
handle
  if Decide () then 1 else 2
with
  | return x      -> x
  | Decide (), k -> k true + k false
```

operation

handler

- Have been adopted in OCaml 5

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()


with h handle (Tick (); Tock ())
```

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (Tick (); Tock ())
```

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (Tick (); Tock ())    ⟶  1 :: with h handle ((); Tock ())
```

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()


with h handle (Tick (); Tock ())      ⟶  1 :: with h handle ((); Tock ())

                                      ⟶  1 :: with h handle (Tock ())
```

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (Tick (); Tock ())    ⟶  1 :: with h handle ((); Tock ())

                                    ⟶  1 :: with h handle (Tock ())
```

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (Tick (); Tock ())    ⟶  1 :: with h handle ((); Tock ())
                                     ⟶  1 :: with h handle (Tock ())
                                     ⟶  1 :: 0 :: with h handle ()
```

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (Tick (); Tock ())    ⟶  1 :: with h handle ((); Tock ())
                                    ⟶  1 :: with h handle (Tock ())
                                    ⟶  1 :: 0 :: with h handle ()
```

Algebraic Effects and Handlers

- **Example**

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (Tick (); Tock ())    ⟶  1 :: with h handle ((); Tock ())
                                    ⟶  1 :: with h handle (Tock ())
                                    ⟶  1 :: 0 :: with h handle ()
                                    ⟶  1 :: 0 :: []
                                    =  [1; 0]
```

Algebraic Effects and Handlers

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

int list

int list

int list

unit → int list

⊢ `with h handle (Tick();Tock())` : int list

Algebraic Effects and Handlers

value types $T ::= B \mid T \to C$
computation types $C ::= \Sigma \rhd T$
signatures $\Sigma ::= \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi}\}_i$

$$\frac{\Gamma \vdash e : \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi}\}_i \rhd T_0 \qquad \Gamma, x : T_0 \vdash e_r : \boldsymbol{C} \qquad (\Gamma, x : T_{ai}, k : T_{bi} \to \boldsymbol{C} \vdash e_i : \boldsymbol{C})_i}{\Gamma \vdash \textbf{with}\ \{x \mapsto e_r, (\mathrm{Op}_i\ x, k \mapsto e_i)_i\}\ \textbf{handle}\ e : \boldsymbol{C}}$$

$$\frac{\Sigma \ni \mathrm{Op} : T_a \twoheadrightarrow T_b \qquad \Gamma \vdash v : T_a}{\Gamma \vdash \mathrm{Op}\ v : \Sigma \rhd T_b}$$

$\Gamma \vdash e : C$

Refinement Types

$$\{x : B \mid \phi\}$$

Formula

Base Type (int, bool etc.)

- Types endowed with predicates
- E.g. $1 + 2 : \{x : \mathrm{int} \mid x = 3\}$
  $1 + 2 : \{x : \mathrm{int} \mid x > 0\}$
- Can represent more specific properties
  - A tool of program verification

Contribution

Refinement type system for algebraic effect handlers

$\vdash$ `with h handle (Tick (); Tock ())` $: \{z : \mathrm{int\ list} \mid z = [1; 0]\}$

- Enables precise verification of algebraic effect handlers
- Can be built on top of existing languages
  - Our implementation for OCaml 5 programs

Challenge

```
let h = handler
| return x -> x
| Tick (), k -> 1 :: k ()
| Tock (), k -> 0 :: k ()
```

The precise types depend on the **order** of the operations

```
with h handle
  (Tick (); Tock ())
(* ==> [1; 0] *)
```

$: \{z\colon \mathrm{int\ list} \mid z = [1;0]\}$

```
with h handle
  (Tock (); Tick ())
(* ==> [0; 1] *)
```

$: \{z\colon \mathrm{int\ list} \mid z = [0;1]\}$

Challenge

```
let h = handler
| return x -> x
| Decide (), k -> k true - k false


with h handle
  (if Decide () then n1 else n2)
(* ==> n1 - n2 *)
```

$: \{z\colon \mathrm{int} \mid z = n_1 - n_2\}$

The precise type depends on
which branch returns **what value**

Challenge

$$\frac{??}{\Gamma \vdash \textbf{with } h \textbf{ handle } e : ??}$$

Challenge

Need to be aware of the **structure** of e
to obtain precise types

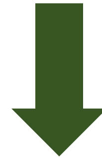
$$\frac{??}{\Gamma \vdash \textbf{with}\ h\ \textbf{handle}\ e : ??}$$

Challenge

Need to be aware of the **structure** of e
to obtain precise types

$$??$$

$$\Gamma \vdash \textbf{with}\ h\ \textbf{handle}\ e : ??$$



We adopt

Answer Type Modification

[Danvy+90; Materzok+11]

Answer Types

- Types of **contexts**
  - = return types of **continuations**
  - = types of **the closest outer handling constructs (delimiters)**

$$\mathcal{E}[\,\textbf{with}\ h\ \textbf{handle}\ \mathcal{F}[e]\,]$$

evaluation context

handler-free evaluation context

Answer type of e = type of $\textbf{with}\ h\ \textbf{handle}\ \mathcal{F}[\,]$

$$\mathcal{F} ::= [\,] \mid \textbf{let}\ x = \mathcal{F}\ \textbf{in}\ e$$
$$\mathcal{E} ::= [\,] \mid \textbf{let}\ x = \mathcal{E}\ \textbf{in}\ e \mid \textbf{with}\ h\ \textbf{handle}\ \mathcal{E}$$

Answer Types in Ordinary Type Systems

value types $T ::= B \mid T \to C$
computation types $C ::= \Sigma \rhd T$
signatures $\Sigma ::= \{\mathrm{Op}_i \colon T_{ai} \twoheadrightarrow T_{bi}\}_i$

answer type

$$\frac{\Gamma \vdash e : \{\mathrm{Op}_i \colon T_{ai} \twoheadrightarrow T_{bi}\}_i \rhd T_0 \qquad \Gamma, x \colon T_0 \vdash e_r : \boldsymbol{C} \qquad (\Gamma, x \colon T_{ai}, k \colon T_{bi} \to \boldsymbol{C} \vdash e_i : \boldsymbol{C})_i}{\Gamma \vdash \mathbf{with}\ \{x \mapsto e_r, (\mathrm{Op}_i\ x, k \mapsto e_i)_i\}\ \mathbf{handle}\ e : \boldsymbol{C}}$$

$$\frac{\Sigma \ni \mathrm{Op} \colon T_a \twoheadrightarrow T_b \qquad \Gamma \vdash v : T_a}{\Gamma \vdash \mathrm{Op}\ v : \Sigma \rhd T_b}$$

$\boxed{\Gamma \vdash e : C}$

Answer Type Modification

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (Tick (); Tock ())
```

Answer Type Modification

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

unit → {z: **int list** | z = [ ]}

`with h handle (   )` returns [ ]

```
with h handle (Tick (); Tock ())
→ 1 :: with h handle ((); Tock ())
→ 1 :: with h handle (Tock ())
→ 1 :: 0 :: with h handle ()
→ 1 :: 0 :: []
```

Answer Type Modification

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

unit → $\{z\colon \textbf{int list} \mid z = [\,]\}$

$\{z\colon \textbf{int list} \mid z = [0]\}$

```
with h handle (      ) returns []
1 ::                   takes [0]
```

```
with h handle (Tick (); Tock ())
→ 1 :: with h handle ((); Tock ())
→ 1 :: with h handle (Tock ())
→ 1 :: 0 :: with h handle ()
→ 1 :: 0 :: []
```

Answer Type Modification

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

Initial answer type

unit → $\{z\colon \text{int list} \mid z = [\,]\}$

Final answer type

$\{z\colon \text{int list} \mid z = [0]\}$

Answer type modification (ATM)

Answer refinement modification (ARM)

```
with h handle (Tick (); Tock ())
→ 1 :: with h handle ((); Tock ())
→ 1 :: with h handle (Tock ())
→ 1 :: 0 :: with h handle ()
→ 1 :: 0 :: []
```

Answer Type Modification

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

unit $\to \{z : \mathbf{int\ list} \mid z = [0]\}$

```
with h handle (Tick (); Tock ())
→ 1 :: with h handle ((); Tock ())
→ 1 :: with h handle (Tock ())
→ 1 :: 0 :: with h handle ()
→ 1 :: 0 :: []
```

Answer Type Modification

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

unit $\to$ $\{z\colon \mathbf{int\ list} \mid z = [0]\}$

$\{z\colon \mathbf{int\ list} \mid z = [1; 0]\}$

```
with h handle (Tick (); Tock ())
→ 1 :: with h handle ((); Tock ())
→ 1 :: with h handle (Tock ())
→ 1 :: 0 :: with h handle ()
→ 1 :: 0 :: []
```

Type Syntax

value types	$T ::= \{x : B \mid \phi\} \mid T \to C$
computation types	$C ::= \Sigma \rhd T \,/\, \boldsymbol{S}$
signatures	$\Sigma ::= \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi} \,/\, \boldsymbol{C_{1i}} \Rightarrow \boldsymbol{C_{2i}}\}_i$
control effects	$\boldsymbol{S} ::= \square \mid \boldsymbol{C_1} \Rightarrow \boldsymbol{C_2}$

Typing Rules

- **Example**

$$\Sigma = \{\ \text{Tick: unit} \twoheadrightarrow \text{unit} \,/\, \{z\text{: int list} \mid z = [0]\} \Rightarrow \{z\text{: int list} \mid z = [1;0]\},$$
$$\text{Tock: unit} \twoheadrightarrow \text{unit} \,/\, \{z\text{: int list} \mid z = [\,]\} \Rightarrow \{z\text{: int list} \mid z = [0]\}\ \}$$

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (
```

`Tick ();` $: \Sigma \rhd \text{unit} \,/\, \{z\text{: int list} \mid z = [0]\} \Rightarrow \{z\text{: int list} \mid z = [1;0]\}$

`Tock ()` $: \Sigma \rhd \text{unit} \,/\, \{z\text{: int list} \mid z = [\,]\} \Rightarrow \{z\text{: int list} \mid z = [0]\}$

```
)
```

Typing Rules

- **Example**

$$\Sigma = \{\, \mathrm{Tick} \colon \mathrm{unit} \twoheadrightarrow \mathrm{unit} \,/\, \{z \colon \mathrm{int\ list} \mid z = [0]\} \Rightarrow \{z \colon \mathrm{int\ list} \mid z = [1; 0]\},$$
$$\mathrm{Tock} \colon \mathrm{unit} \twoheadrightarrow \mathrm{unit} \,/\, \{z \colon \mathrm{int\ list} \mid z = [\,]\} \Rightarrow \{z \colon \mathrm{int\ list} \mid z = [0]\} \,\}$$

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()

with h handle (
  Tick ();
  Tock ()
)
```

$$: \Sigma \rhd \mathrm{unit} \,/\, \{z \colon \mathrm{int\ list} \mid z = [\,]\} \Rightarrow \{z \colon \mathrm{int\ list} \mid z = [1; 0]\}$$

Typing Rules

- **Example**

$$\Sigma = \left\{ \begin{array}{l} \text{Tick}: \text{unit} \twoheadrightarrow \text{unit} \,/\, \{z: \text{int list} \mid z = [0]\} \Rightarrow \{z: \text{int list} \mid z = [1;0]\}, \\ \text{Tock}: \text{unit} \twoheadrightarrow \text{unit} \,/\, \{z: \text{int list} \mid z = [\,]\} \Rightarrow \{z: \text{int list} \mid z = [0]\} \end{array} \right\}$$

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()
```

```
with h handle (

  Tick ();

  Tock ()

)
```

$: \{z: \text{int list} \mid z = [1;0]\}$

Typing Rules

$$\frac{\Gamma \vdash e : \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi} \;/\; C_{1i} \Rightarrow C_{2i}\}_i \rhd T_0 \;/\; C_1 \Rightarrow C_2 \qquad \Gamma, x : T_0 \vdash e_r : C_1 \qquad (\Gamma, x : T_{ai}, k : T_{bi} \rightarrow C_{1i} \vdash e_i : C_{2i})_i}{\Gamma \vdash \textbf{with}\ \{x \mapsto e_r, (\mathrm{Op}_i\ x, k \mapsto e_i)_i\}\ \textbf{handle}\ e : C_2}$$

$$\frac{\Sigma \ni \mathrm{Op} : T_a \twoheadrightarrow T_b \;/\; C_1 \Rightarrow C_2 \qquad \Gamma \vdash v : T_a}{\Gamma \vdash \mathrm{Op}\ v : \Sigma \rhd T_b \;/\; C_1 \Rightarrow C_2}$$

$\boxed{\Gamma \vdash e : C}$

Typing Rules

$$\frac{\Gamma \vdash e : \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi} \;/\; C_{1i} \Rightarrow C_{2i}\}_i \rhd T_0 \;/\; C_1 \Rightarrow C_2 \qquad \Gamma, x : T_0 \vdash e_r : C_1 \quad (\Gamma, x : T_{ai}, k : T_{bi} \to C_{1i} \vdash e_i : C_{2i})_i}{\Gamma \vdash \textbf{with}\; \{x \mapsto e_r, (\mathrm{Op}_i\; x, k \mapsto e_i)_i\}\; \textbf{handle}\; e : C_2}$$

$$\frac{\Sigma \ni \mathrm{Op} : T_a \twoheadrightarrow T_b \;/\; C_1 \Rightarrow C_2 \qquad \Gamma \vdash v : T_a}{\Gamma \vdash \mathrm{Op}\; v : \Sigma \rhd T_b \;/\; C_1 \Rightarrow C_2}$$

$\Gamma \vdash e : C$

Typing Rules

$$\frac{\Gamma \vdash e : \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi} \;/\; C_{1i} \Rightarrow C_{2i}\}_i \rhd T_0 \;/\; C_1 \Rightarrow C_2 \qquad \Gamma, x : T_0 \vdash e_r : C_1 \quad (\Gamma, x : T_{ai}, k : T_{bi} \to C_{1i} \vdash e_i : C_{2i})_i}{\Gamma \vdash \textbf{with}\ \{x \mapsto e_r, (\mathrm{Op}_i\ x, k \mapsto e_i)_i\}\ \textbf{handle}\ e : C_2}$$

$$\frac{\Sigma \ni \mathrm{Op} : T_a \twoheadrightarrow T_b \;/\; C_1 \Rightarrow C_2 \qquad \Gamma \vdash v : T_a}{\Gamma \vdash \mathrm{Op}\ v : \Sigma \rhd T_b \;/\; C_1 \Rightarrow C_2}$$

$\boxed{\Gamma \vdash e : C}$

In the Paper

- A more sophisticated type system and the proof of the type safety
  - $\Sigma ::= \{\mathrm{Op}_i : \forall X : \bar{B}. x : T_{ai} \twoheadrightarrow T_{bi} \mathbin{/} y. C_{1i} \Rightarrow C_{2i}\}_i$
    - Predicate polymorphism
    - Argument-dependent continuation types
- Implementation
  - On top of OCaml 5.0
  - Experiments on 50+ examples

https://github.com/hiroshi-unno/coar

- Another way of verification via CPS transformation
  - Removal of handlers + verification with existing refinement type systems
  - Bidirectionally type-preserving CPS transformation
    - Needs type annotations on source programs

Appendix

Extension 1: Dependent Types for Continuations

value types $T ::= B \mid T \to C$
computation types $C ::= \Sigma \rhd T \,/\, S$
signatures $\Sigma ::= \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi} \,/\, y.\, C_{1i} \Rightarrow C_{2i}\}_i$
control effects $S ::= \square \mid y.\, C_1 \Rightarrow C_2$

$$\frac{\Gamma \vdash e : \{\mathrm{Op}_i : T_{ai} \twoheadrightarrow T_{bi} \,/\, y.\, C_{1i} \Rightarrow C_{2i}\}_i \rhd T_0 \,/\, C_1 \Rightarrow C_2 \quad \Gamma, x : T_0 \vdash e_r : C_1 \quad (\Gamma, x : T_{ai}, k : (y : T_{bi}) \to C_{1i} \vdash e_i : C_{2i})_i}{\Gamma \vdash \mathbf{with}\ \{x \mapsto e_r, (\mathrm{Op}_i\ x, k \mapsto e_i)_i\}\ \mathbf{handle}\ e : C_2}$$

$$\frac{\Sigma \ni \mathrm{Op} : T_a \twoheadrightarrow T_b \,/\, y.\, C_1 \Rightarrow C_2 \qquad \Gamma \vdash v : T_a}{\Gamma \vdash \mathrm{Op}\ v : \Sigma \rhd (T_b \,/\, y.\, C_1 \Rightarrow C_2)}$$

$\boxed{\Gamma \vdash e : C}$

Extension 1: Dependent Types for Continuations

- **Example**

```
let h = handler
| return x -> x
| Decide (), k -> k true - k false
```

$b\colon \mathrm{bool} \to \{z\colon \mathrm{int} \mid b \Rightarrow z = n_1 \land \neg b \Rightarrow z = n_2\}$

$b \Rightarrow z = n_1 \land \neg b \Rightarrow z = n_2\}$

$\Rightarrow \{z\colon \mathrm{int} \mid z = n_1 - n_2\}\}$

```
with h handle (
  let b = Decide () in
  if b then n1 else n2
)
```

$\{z\colon \mathrm{int} \mid z = n_1 - n_2\}$

$\{z\colon \mathrm{int} \mid b \Rightarrow z = n_1 \land \neg b \Rightarrow z = n_2\}$

Extension 2: Predicate Polymorphism

value types	$T ::= B \mid T \to C$
computation types	$C ::= \Sigma \rhd T \,/\, S$
signatures	$\Sigma ::= \{\mathrm{Op}_i : \forall X{:}\bar{B}.\, T_{ai} \twoheadrightarrow T_{bi} \,/\, C_{1i} \Rightarrow C_{2i}\}_i$
control effects	$S ::= \square \mid C_1 \Rightarrow C_2$

$$\frac{\Gamma \vdash e : \{\mathrm{Op}_i : \forall X{:}\bar{B}.\, T_{ai} \twoheadrightarrow T_{bi} \,/\, C_{1i} \Rightarrow C_{2i}\}_i \rhd T_0 \,/\, C_1 \Rightarrow C_2 \qquad \Gamma, x{:}T_0 \vdash e_r : C_1 \quad (\Gamma, X{:}\bar{B}, x{:}T_{ai}, k{:}T_{bi} \to C_{1i} \vdash e_i : C_{2i})_i}{\Gamma \vdash \textbf{with}\ \{x \mapsto e_r, (\mathrm{Op}_i\ x, k \mapsto e_i)_i\}\ \textbf{handle}\ e : C_2}$$

$$\frac{\Sigma \ni \mathrm{Op} : \forall X{:}\bar{B}.\, T_a \twoheadrightarrow T_b \,/\, C_1 \Rightarrow C_2 \qquad \Gamma \vdash v : T_a \qquad \Gamma \vdash A : \bar{B}}{\Gamma \vdash \mathrm{Op}\ v : \Sigma \rhd (T_b \,/\, C_1 \Rightarrow C_2)[A/X]}$$

$\boxed{\Gamma \vdash e : C}$

Extension 2: Predicate Polymorphism

- **Example**

$$\Sigma = \{\ \mathrm{Tick}\colon \forall X(\mathrm{int\ list}).\ \mathrm{unit} \twoheadrightarrow \mathrm{unit}\ /\ \{z\colon \mathrm{int\ list} \mid X(z)\} \Rightarrow \{z\colon \mathrm{int\ list} \mid \forall l. X(l) \Rightarrow z = 1 :: l\},\ \mathrm{Tock}\colon \forall Y(\mathrm{int\ list}).\ \mathrm{unit} \twoheadrightarrow \mathrm{unit}\ /\ \{z\colon \mathrm{int\ list} \mid Y(z)\} \Rightarrow \{z\colon \mathrm{int\ list} \mid \forall l. Y(l) \Rightarrow z = 0 :: l\}\ \}$$

```
let h = handler
  | return x -> []
  | Tick (), k -> 1 :: k ()
  | Tock (), k -> 0 :: k ()

with h handle (
  Tick ();
  Tock ();
  Tick ();
  Tock ()
)
```

$\{z\colon \mathrm{int\ list} \mid z = [1;0;1;0]\}$

$X \mapsto \lambda z. z = [0;1;0]$

$\{z\colon \mathrm{int\ list} \mid z = [0;1;0]\}$

$Y \mapsto \lambda z. z = [1;0]$

$\{z\colon \mathrm{int\ list} \mid z = [1;0]\}$

$X \mapsto \lambda z. z = [0]$

$\{z\colon \mathrm{int\ list} \mid z = [0]\}$

$Y \mapsto \lambda z. z = [\,]$

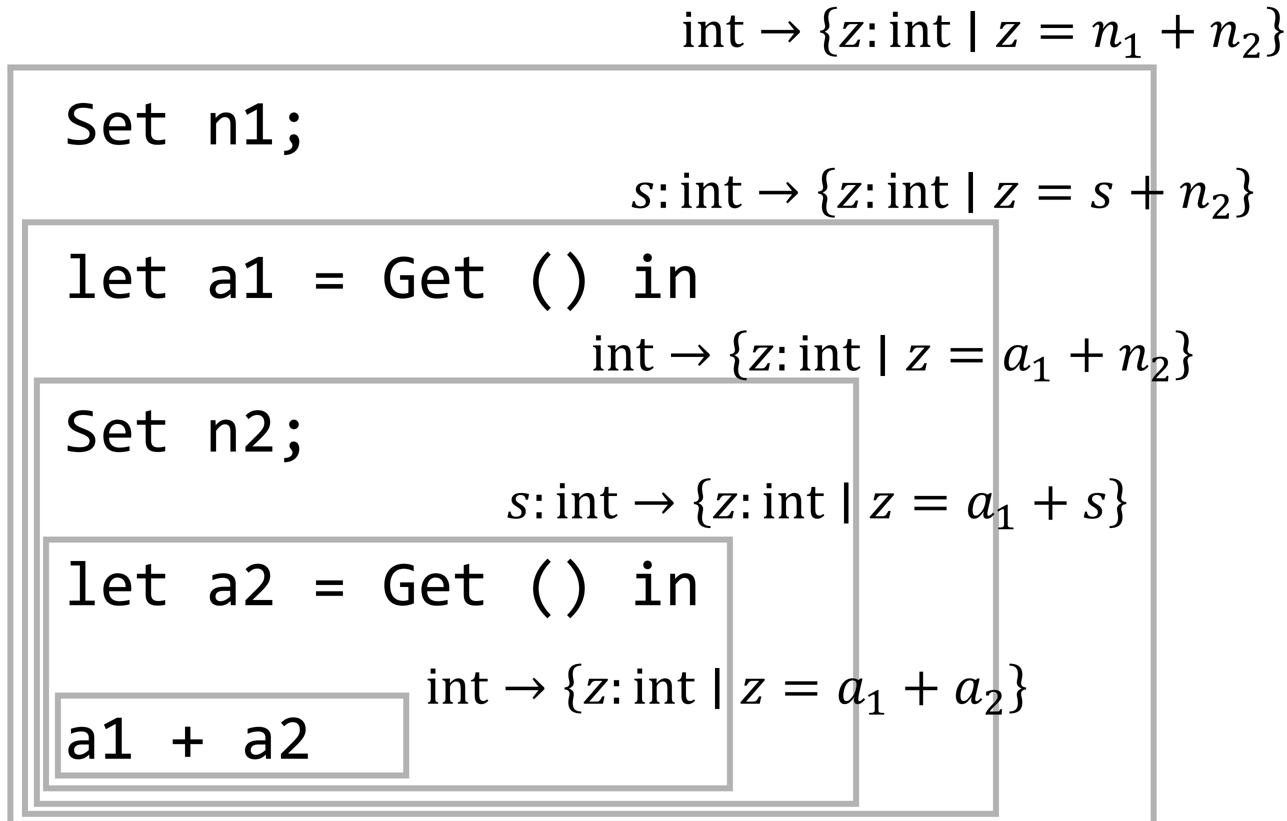
$\{z\colon \mathrm{int\ list} \mid z = [\,]\}$

Extension 2: Predicate Polymorphism

- **Example**

```
let h = handler
| return x  -> λs. x
| Set  v, k -> λs. k () v
| Get (), k -> λs. k  s s
```

```
with h handle
```



$$\Sigma = \Big\{\text{Set}{:}\ \forall X{:}\,(\text{int},\text{int}).\ x{:}\,\text{int} \twoheadrightarrow \text{unit}\ /\ s{:}\,\text{int} \to \{z{:}\,\text{int} \mid X(z,s)\} \Rightarrow \text{int} \to \{z{:}\,\text{int} \mid X(z,x)\},$$
$$\text{Get}{:}\ \forall Y{:}\,(\text{int},\text{int},\text{int}).\ \text{unit} \twoheadrightarrow \text{int}\ /\ y.\ s{:}\,\text{int} \to \{z{:}\,\text{int} \mid Y(z,s,y)\} \Rightarrow s'{:}\,\text{int} \to \{z{:}\,\text{int} \mid Y(z,s',s')\}\Big\}$$